

Write your name here					
Surname			Other names		
Pearson		Centre Number		Candidate Number	
Edexcel GCE		<input style="width: 20px; height: 20px;" type="text"/>		<input style="width: 20px; height: 20px;" type="text"/>	
AS and A level Further Mathematics Further Pure Mathematics 1					
Practice Paper Parabolas, ellipses and hyperbolas					
You must have: Mathematical Formulae and Statistical Tables (Pink)					Total Marks <input style="width: 50px; height: 30px;" type="text"/>

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
- There are 11 questions in this question paper. The total mark for this paper is 100.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. A parabola C has cartesian equation $y^2 = 16x$. The point $P(4t^2, 8t)$ is a general point on C .

(a) Write down the coordinates of the focus F and the equation of the directrix of C .

(3)

(b) Show that the equation of the normal to C at P is $y + tx = 8t + 4t^3$.

(5)

(Total 8 marks)

2. A parabola C has equation $y^2 = 4ax$, $a > 0$

The points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ lie on C , where $p \neq 0$, $q \neq 0$, $p \neq q$.

(a) Show that an equation of the tangent to the parabola at P is

$$py - x = ap^2$$

(4)

(b) Write down the equation of the tangent at Q .

(1)

The tangent at P meets the tangent at Q at the point R .

(c) Find, in terms of p and q , the coordinates of R , giving your answers in their simplest form.

(4)

Given that R lies on the directrix of C ,

(d) find the value of pq .

(2)

(Total 11 marks)

3.

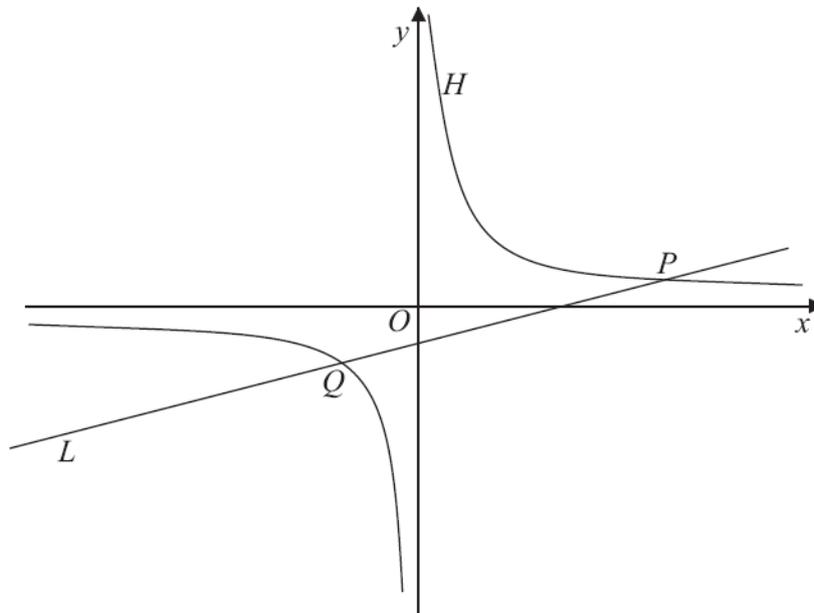


Figure 1

Figure 1 shows a rectangular hyperbola H with parametric equations

$$x = 3t, \quad y = \frac{3}{t}, \quad t \neq 0$$

The line L with equation $6y = 4x - 15$ intersects H at the point P and at the point Q as shown in Figure 1.

(a) Show that L intersects H where $4t^2 - 5t - 6 = 0$.

(3)

(b) Hence, or otherwise, find the coordinates of points P and Q .

(5)

(Total 8 marks)

4.

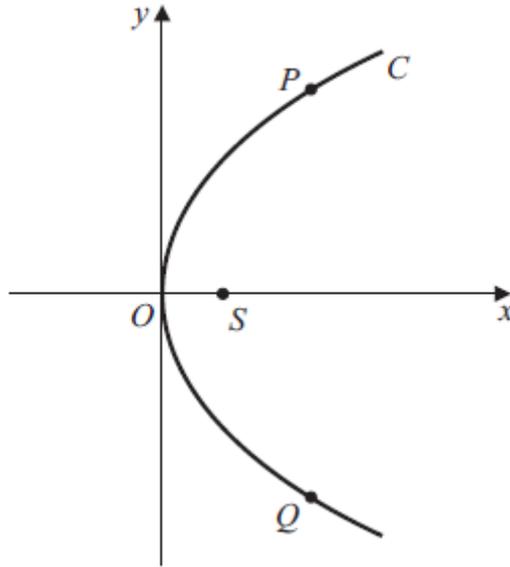


Figure 2

Figure 2 shows a sketch of the parabola C with equation $y^2 = 8x$.

The point P lies on C , where $y > 0$, and the point Q lies on C , where $y < 0$.

The line segment PQ is parallel to the y -axis.

Given that the distance PQ is 12,

(a) write down the y -coordinate of P ,

(1)

(b) find the x -coordinate of P .

(2)

Figure 2 shows the point S which is the focus of C .

The line l passes through the point P and the point S .

(c) Find an equation for l in the form $ax + by + c = 0$, where a , b and c are integers.

(4)

(Total 7 marks)

5.

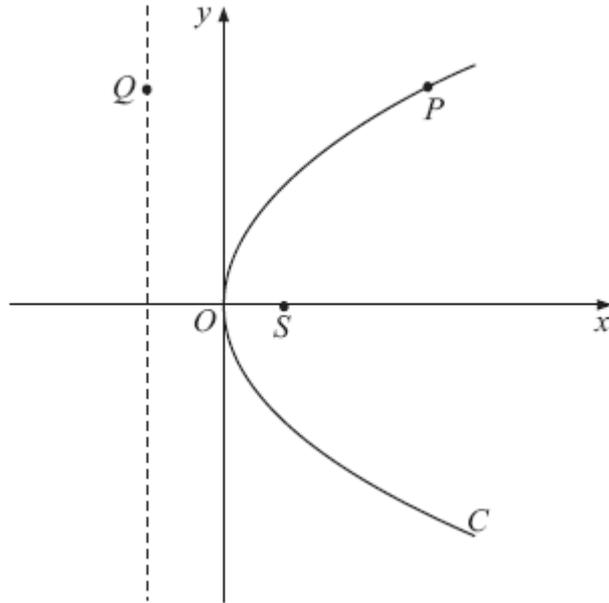


Figure 3

Figure 3 shows a sketch of the parabola C with equation $y^2 = 36x$.

The point S is the focus of C .

(a) Find the coordinates of S .

(1)

(b) Write down the equation of the directrix of C .

(1)

Figure 3 shows the point P which lies on C , where $y > 0$, and the point Q which lies on the directrix of C . The line segment QP is parallel to the x -axis.

Given that the distance PS is 25,

(c) write down the distance QP ,

(1)

(d) find the coordinates of P ,

(3)

(e) find the area of the trapezium $OSPQ$.

(2)

(Total 8 marks)

6. The parabola C has equation $y^2 = 4ax$, where a is a positive constant.

The point $P(at^2, 2at)$ is a general point on C .

- (a) Show that the equation of the tangent to C at $P(at^2, 2at)$ is

$$ty = x + at^2$$

(4)

The tangent to C at P meets the y -axis at a point Q .

- (b) Find the coordinates of Q .

(1)

Given that the point S is the focus of C ,

- (c) show that PQ is perpendicular to SQ .

(3)

(Total 8 marks)

7. The rectangular hyperbola, H , has cartesian equation $xy = 25$.

The point $P\left(5p, \frac{5}{p}\right)$ and the point $Q\left(5q, \frac{5}{q}\right)$, where $p, q \neq 0$, $p \neq q$, are points on the rectangular hyperbola H .

- (a) Show that the equation of the tangent at point P is

$$p^2y + x = 10p.$$

(4)

- (b) Write down the equation of the tangent at point Q .

(1)

The tangents at P and Q meet at the point N .

Given $p + q \neq 0$,

- (c) show that point N has coordinates $\left(\frac{10pq}{p+q}, \frac{10}{p+q}\right)$.

(4)

The line joining N to the origin is perpendicular to the line PQ .

- (d) Find the value of p^2q^2 .

(5)

(Total 14 marks)

8. The rectangular hyperbola H has equation $xy = c^2$, where c is a positive constant.

The point $P\left(ct, \frac{c}{t}\right)$, $t \neq 0$, is a general point on H .

- (a) Show that an equation for the tangent to H at P is

$$x + t^2 y = 2ct.$$

(4)

The tangent to H at the point P meets the x -axis at the point A and the y -axis at the point B .

Given that the area of the triangle OAB , where O is the origin, is 36,

- (b) find the exact value of c , expressing your answer in the form $k\sqrt{2}$, where k is an integer.

(4)

(Total 8 marks)

9. The parabola C has equation $y^2 = 48x$.

The point $P(12t^2, 24t)$ is a general point on C .

- (a) Find the equation of the directrix of C .

(2)

- (b) Show that the equation of the tangent to C at $P(12t^2, 24t)$ is

$$x - ty + 12t^2 = 0.$$

(4)

The tangent to C at the point $(3, 12)$ meets the directrix of C at the point X .

- (c) Find the coordinates of X .

(4)

(Total 10 marks)

10.

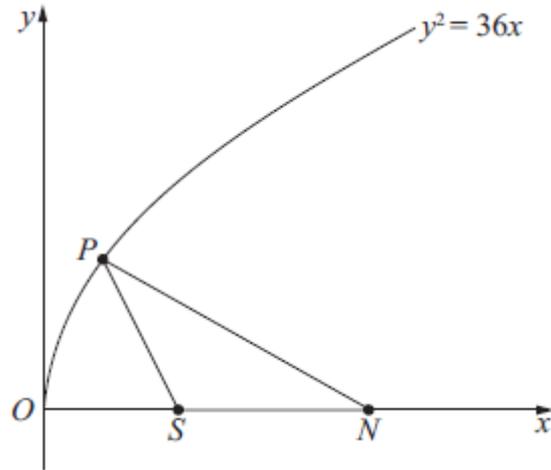


Figure 4

Figure 4 shows a sketch of part of the parabola with equation $y^2 = 36x$.

The point $P(4, 12)$ lies on the parabola.

(a) Find an equation for the normal to the parabola at P .

(5)

This normal meets the x -axis at the point N and S is the focus of the parabola, as shown in Figure 4.

(b) Find the area of triangle PSN .

(4)

(Total 9 marks)

11. The rectangular hyperbola H has cartesian equation $xy = 9$.

The points $P\left(3p, \frac{3}{p}\right)$ and $Q\left(3q, \frac{3}{q}\right)$ lie on H , where $p \neq \pm q$.

(a) Show that the equation of the tangent at P is $x + p^2y = 6p$.

(4)

(b) Write down the equation of the tangent at Q .

(1)

The tangent at the point P and the tangent at the point Q intersect at R .

(c) Find, as single fractions in their simplest form, the coordinates of R in terms of p and q .

(4)

(Total 9 marks)

TOTAL FOR PAPER: 100 MARKS